Golden Section Transform:
Golden Mean of Golden Ratios
Preview

Jun Li

http://GoldenSectionTransform.com/

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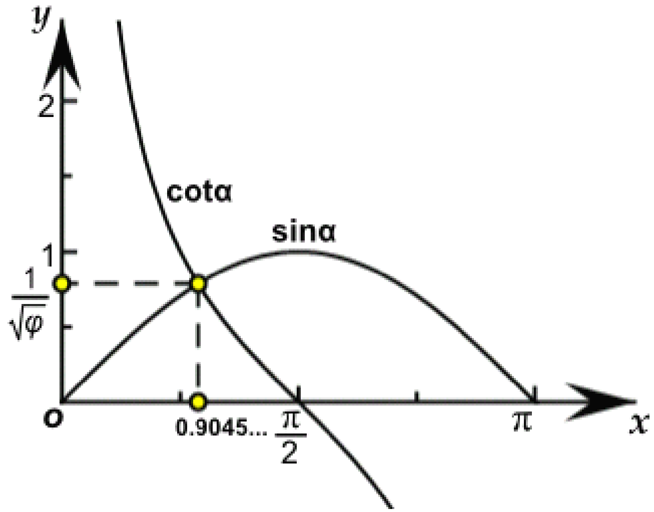
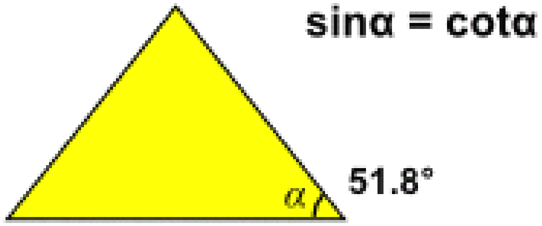
Reverse Order Low Golden Section Transform (RLGST)

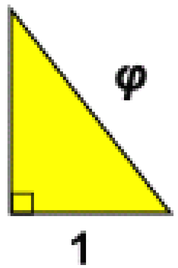
Johannes Kepler

http://en.wikipedia.org/wiki/Johannes_Kepler





Kepler triangle



RLGST Lifting Scheme (Jun Li):

Let $F_1 = 1,\ F_2 = 1,\ \{F_l\} = 1, 1, 2, 3, 5, 8, \ldots$ Let $\{X_u\}$ be the original sequence, let $\{S_v\}$ be a sequence as the "Low" part of reverse order low golden section transform(RLGST) of $\{X_u\}$, let $\{D_w\}$ be a sequence as the "High" part of RLGST of $\{X_u\}$, we have $\{X_u\} = X_1, X_2, \cdots, X_u; \{S_v\} = S_1, S_2, \cdots, S_v; \{D_w\} = D_1, D_2, \cdots, D_w$ where $u = F_{l+2},\ v = F_{l+1},\ w = F_l\ (l \geq 1, l \in \mathbb{Z})$

The lifting scheme of normalized reverse order low golden section transform(RLGST) is:

$$\begin{cases} {}^0S^k_{L_n} = X_{U_n-1},\ {}^0D^k_n = X_{U_n} \\ {}^1D^k_n = {}^0S^k_{L_n} - \sqrt{\frac{F_{k+1}}{F_k}} \cdot {}^0D^k_n,\ {}^1S^k_{L_n} = {}^0D^k_n + \frac{\sqrt{F_k F_{k+1}}}{F_{k+2}} \cdot {}^1D^k_n \\ S_{L_n} = \sqrt{\frac{F_{k+2}}{F_k}} \cdot {}^1S^k_{L_n},\ S_{U_m} = X_{L_m+U_m},\ D_n = {}^1D^k_n \cdot \sqrt{\frac{F_k}{F_{k+2}}} \end{cases}$$

Jun Li

and the reconstruction algorithm is:

$$\begin{cases} {}^1D^k_n = \sqrt{\frac{F_{k+2}}{F_k}} \cdot D_n,\ {}^1S^k_{L_n} = S_{L_n} \cdot \sqrt{\frac{F_k}{F_{k+2}}} \\ {}^0D^k_n = {}^1S^k_{L_n} - \frac{\sqrt{F_k F_{k+1}}}{F_{k+2}} \cdot {}^1D^k_n,\ {}^0S^k_{L_n} = {}^1D^k_n + \sqrt{\frac{F_{k+1}}{F_k}} \cdot {}^0D^k_n \\ X_{U_n} = {}^0D^k_n,\ X_{L_m+U_m} = S_{U_m},\ X_{U_n-1} = {}^0S^k_{L_n} \end{cases}$$

where

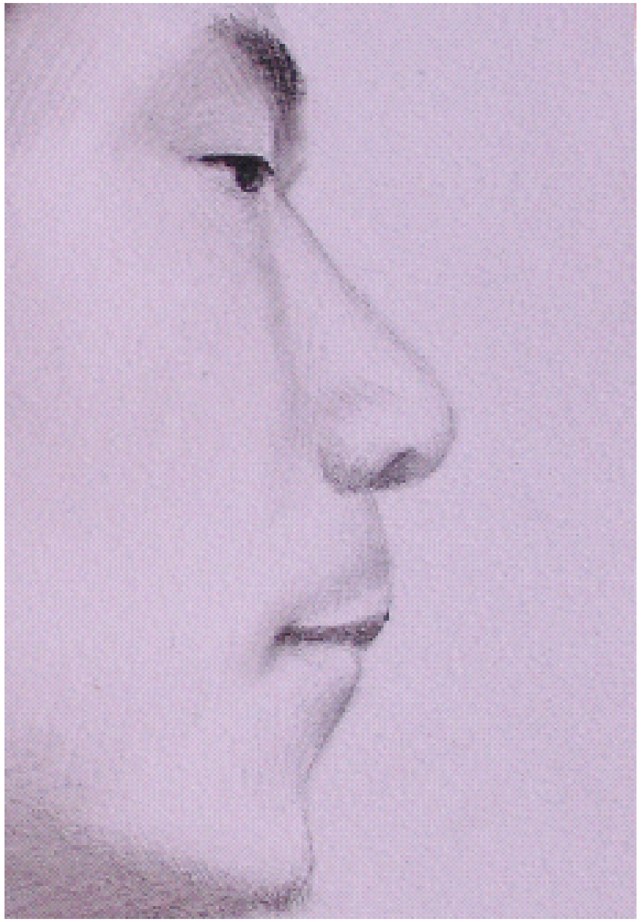
$L_n = \left\lfloor \frac{1+\sqrt{5}}{2} n \right\rfloor, \{L_n\} = 1, 3, 4, 6, 8, \cdots$ Wythoff lower sequence A000201 in OEIS

$U_n = \left\lfloor \frac{3+\sqrt{5}}{2} n \right\rfloor, \{U_n\} = 2, 5, 7, 10, 13, \cdots$ Wythoff upper sequence A001950 in OEIS

where

$m = 1, 2, 3, 4, \cdots, F_{l-1};\ n = 1, 2, 3, 4, \cdots, F_l;\ level\ k = 1, 2, 3, 4, \cdots, l (k \leq l; m, n, k \in \mathbb{N}^*)$

Reverse Order High Golden Section Transform (RHGST)



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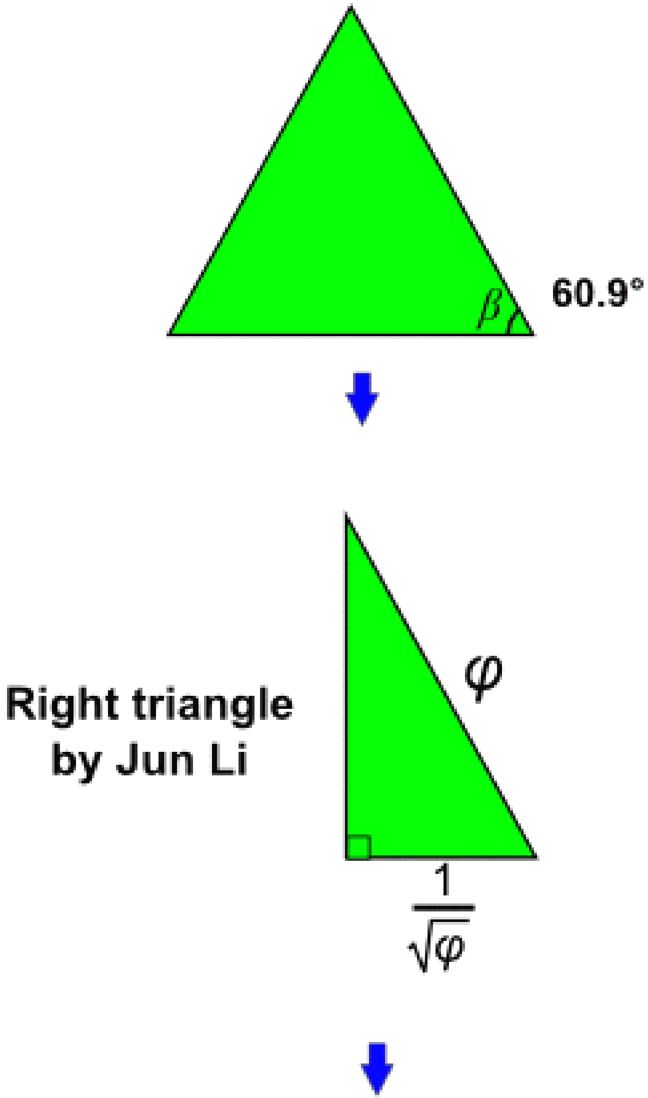

RHGST Lifting Scheme (Jun Li):

Let $F_1 = 1,\ F_2 = 1,\ \{F_l\} = 1, 1, 2, 3, 5, 8, \ldots$ Let $\{X_u\}$ be the original sequence, let $\{S_v\}$ be a sequence as the "Low" part of reverse order high golden section transform of $\{X_u\}$, let $\{D_w\}$ be a sequence as the "High" part of RHGST of $\{X_u\}$, we have $\{X_u\} = X_1, X_2, \cdots, X_u; \{S_v\} = S_1, S_2, \cdots, S_v; \{D_w\} = D_1, D_2, \cdots, D_w$ where $u = F_{l+2},\ v = F_l,\ w = F_{l+1}\ (l \geq 1, l \in \mathbb{Z})$

The lifting scheme of normalized reverse order high golden section transform(RHGST) is:

when $l = 1$

$$\begin{cases} {}^0S_1^k = X_1,\ {}^0D_1^k = X_2 \\ {}^1D_1^k = {}^0S_1^k - \sqrt{\frac{F_{2k}}{F_{2k-1}}} \cdot {}^0D_1^k,\ {}^1S_1^k = {}^0D_1^k + \frac{\sqrt{F_{2k-1}F_{2k}}}{F_{2k+1}} \cdot {}^1D_1^k \\ S_1 = \sqrt{\frac{F_{2k+1}}{F_{2k-1}}} \cdot {}^1S_1^k,\ D_1 = {}^1D_1^k \cdot \sqrt{\frac{F_{2k-1}}{F_{2k+1}}} \end{cases}$$

Jun Li

when $l \geq 2$

$$\begin{cases} {}^0S_{L_n}^k = X_{L_n+U_n-2},\ {}^0D_{U_n}^k = X_{L_n+U_n} \\ {}^1D_{U_n}^k = {}^0S_{L_n}^k - {}^0D_{U_n}^k,\ {}^1S_{L_n}^k = {}^0D_{U_n}^k + \frac{1}{2} \cdot {}^1D_{U_n}^k \\ {}^2S_{L_n}^k = \sqrt{2} \cdot {}^1S_{L_n}^k,\ {}^0D_{U_n-1}^k = X_{L_n+U_n-1} \\ {}^1D_{U_n-1}^k = {}^2S_{L_n}^k - \sqrt{\frac{2F_{2k}}{F_{2k-1}}} \cdot {}^0D_{U_n-1}^k,\ {}^3S_{L_n}^k = {}^0D_{U_n-1}^k + \frac{\sqrt{2F_{2k-1}F_{2k}}}{F_{2k+2}} \cdot {}^1D_{U_n-1}^k \\ S_{L_n} = \sqrt{\frac{F_{2k+2}}{F_{2k-1}}} \cdot {}^3S_{L_n}^k,\ D_{U_n-1} = {}^1D_{U_n-1}^k \cdot \sqrt{\frac{F_{2k-1}}{F_{2k+2}}},\ D_{U_n} = {}^1D_{U_n}^k / \sqrt{2} \end{cases}$$

when $l \geq 3$

$$\begin{cases} {}^0S_{U_m}^k = X_{L_m+2U_m-1},\ {}^0D_{L_m+U_m}^k = X_{L_m+2U_m} \\ {}^1D_{L_m+U_m}^k = {}^0S_{U_m}^k - \sqrt{\frac{F_{2k}}{F_{2k-1}}} \cdot {}^0D_{L_m+U_m}^k,\ {}^1S_{U_m}^k = {}^0D_{L_m+U_m}^k + \frac{\sqrt{F_{2k-1}F_{2k}}}{F_{2k+1}} \cdot {}^1D_{L_m+U_m}^k \\ S_{U_m} = \sqrt{\frac{F_{2k+1}}{F_{2k-1}}} \cdot {}^1S_{U_m}^k,\ D_{L_m+U_m} = {}^1D_{L_m+U_m}^k \cdot \sqrt{\frac{F_{2k-1}}{F_{2k+1}}} \end{cases}$$

and the reconstruction algorithm is:

when $l = 1$

$$\begin{cases} {}^1D_1^k = \sqrt{\frac{F_{2k+1}}{F_{2k-1}}} \cdot D_1, \ {}^1S_1^k = S_1 \cdot \sqrt{\frac{F_{2k-1}}{F_{2k+1}}} \\ {}^0D_1^k = {}^1S_1^k - \frac{\sqrt{F_{2k-1}F_{2k}}}{F_{2k+1}} \cdot {}^1D_1^k, \ {}^0S_1^k = {}^1D_1^k + \sqrt{\frac{F_{2k}}{F_{2k-1}}} \cdot {}^0D_1^k \\ X_2 = {}^0D_1^k, \ X_1 = {}^0S_1^k \end{cases}$$

when $l \geq 2$

$$\begin{cases} {}^1D_{U_n}^k = \sqrt{2} \cdot D_{U_n}, \ {}^1D_{U_n-1}^k = \sqrt{\frac{F_{2k+2}}{F_{2k-1}}} \cdot D_{U_n-1}, \ {}^3S_{L_n}^k = S_{L_n} \cdot \sqrt{\frac{F_{2k-1}}{F_{2k+2}}} \\ {}^0D_{U_n-1}^k = {}^3S_{L_n}^k - \frac{\sqrt{2F_{2k-1}F_{2k}}}{F_{2k+2}} \cdot {}^1D_{U_n-1}^k, \ {}^2S_{L_n}^k = {}^1D_{U_n-1}^k + \sqrt{\frac{2F_{2k}}{F_{2k-1}}} \cdot {}^0D_{U_n-1}^k \\ X_{L_n+U_n-1} = {}^0D_{U_n-1}^k, \ {}^1S_{L_n}^k = {}^2S_{L_n}^k / \sqrt{2} \\ {}^0D_{U_n}^k = {}^1S_{L_n}^k - \frac{1}{2} \cdot {}^1D_{U_n}^k, \ {}^0S_{L_n}^k = {}^1D_{U_n}^k + {}^0D_{U_n}^k \\ X_{L_n+U_n} = {}^0D_{U_n}^k, \ X_{L_n+U_n-2} = {}^0S_{L_n}^k \end{cases}$$

Jun Li

when $l \geq 3$

$$\begin{cases} {}^1D_{L_m+U_m}^k = \sqrt{\frac{F_{2k+1}}{F_{2k-1}}} \cdot D_{L_m+U_m}, \ {}^1S_{U_m}^k = S_{U_m} \cdot \sqrt{\frac{F_{2k-1}}{F_{2k+1}}} \\ {}^0D_{L_m+U_m}^k = {}^1S_{U_m}^k - \frac{\sqrt{F_{2k-1}F_{2k}}}{F_{2k+1}} \cdot {}^1D_{L_m+U_m}^k, \ {}^0S_{U_m}^k = {}^1D_{L_m+U_m}^k + \sqrt{\frac{F_{2k}}{F_{2k-1}}} \cdot {}^0D_{L_m+U_m}^k \\ X_{L_m+2U_m} = {}^0D_{L_m+U_m}^k, \ X_{L_m+2U_m-1} = {}^0S_{U_m}^k \end{cases}$$

where

$L_n = \left\lfloor \frac{1+\sqrt{5}}{2} n \right\rfloor, \{L_n\} = 1, 3, 4, 6, 8, \cdots$ Wythoff lower sequence A000201 in OEIS

$U_n = \left\lfloor \frac{3+\sqrt{5}}{2} n \right\rfloor, \{U_n\} = 2, 5, 7, 10, 13, \cdots$ Wythoff upper sequence A001950 in OEIS

where

$$m = 1, 2, 3, 4, \cdots, F_{l-2}; \ n = 1, 2, 3, 4, \cdots, F_{l-1}; \ level \ k = 1, 2, 3, \cdots, \left\lfloor \frac{l+1}{2} \right\rfloor (m, n, k \in \mathbb{N}^*)$$

Links

- https://github.com/jasonli30s/
- http://www.mathworks.com/matlabcentral/profile/authors/6361386-jun-li
- http://www.slideshare.net/jasonli1880

- http://en.wikipedia.org/wiki/Great_Pyramid_of_Giza
- http://en.wikipedia.org/wiki/Kepler_triangle
- http://blog.world-mysteries.com/science/the-great-pyramid-and-the-speed-of-light/
- http://www.wanttoknow.nl/hoofdartikelen/mysterieuze-krachten-van-piramide-structuren/
- http://ifdawn.com/esa/tipharet.htm
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- http://en.wikipedia.org/wiki/Fibonacci_word
- http://mathworld.wolfram.com/RabbitSequence.html
- The infinite Fibonacci word, https://oeis.org/A005614
- Lower Wythoff sequence, http://oeis.org/A000201
- Upper Wythoff sequence, http://oeis.org/A001950

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